APPLICATION OF THE TURBIDITY RATIO METHOD IN THE SPHERICAL PARTICLE SIZE DETERMINATION IN THE RANGE $m \in \langle 1.001, 1.315 \rangle$ AND $\alpha \in \langle 0.05, 100 \rangle$*

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Basic scattering functions were used in a novel calculation of the turbidity ratios for particles having the relative refractive index $m = 1{\cdot}001$, $1{\cdot}005$ $(0{\cdot}005)$ $1{\cdot}315$ and the size $\alpha = 0{\cdot}05$ $(0{\cdot}05)$ $6{\cdot}00$ $(0{\cdot}10)$ $15{\cdot}00$ $(0{\cdot}50)$ $70{\cdot}00$ $(1{\cdot}00)$ 100, where $\alpha = \pi L/\lambda$, L is the diameter of the spherical particle, $\lambda = \Lambda/\mu_1$ is the wavelength of light in a medium with the refractive index μ_1 and Λ is the wavelength of light *in vacuo*. The data are tabulated for the wavelength $\lambda = 546{\cdot}1/\mu_w = 409{\cdot}357$ nm, where μ_w is the refractive index of water. A procedure has been suggested how to extend the applicability of Tables to various refractive indices of the medium and to various turbidity ratios τ_a/τ_b obtained with the individual pairs of wavelengths λ_a and λ_b. The selection of these pairs is bound to the sequence condition $\lambda^a = \lambda_0 \varkappa^a$ and $\lambda_b = \lambda_0 \varkappa^b$, in which $b - a = \delta = 1, 2, 3$; $a = -2, -1, 0, 1, 2, \ldots$, $b = a + \delta = -1, 0, 1, 2, \ldots$; $\lambda_0 = \lambda_{a=0} = 326{\cdot}675$ nm; $\varkappa = 546{\cdot}1 : 435{\cdot}8 = 1{\cdot}2531$ is the quotient of the given sequence.

More than ten years ago, one of the authors (B.S.) reported an analysis of the applicability of the turbidity ratio method in the particle size determination[1-3] of polystyrene latexes[2] (relative refractive index $m > 1{\cdot}00$) and of turbid gels[3], or more exactly, of particles arising by the microseparation of diluent in the gel ($m < 1{\cdot}00$). The tabulation of scattering functions and the applicability test of the method were based on data calculated with manual calculators by using approximative methods described by Heller and coworkers[4,5]. Some interpolation procedures employed in the calculations were rather rough, so that the resulting tabulated functions allowed us only an approximative particle size estimation. This is particularly true for the turbid gels under study, for which it was assumed[3] that data tabulated for $m = 1{\cdot}00 + \Delta m$ were also applicable for $m = 1{\cdot}00 - \Delta m$ due to an approximate symmetry of scattering functions about the function values for $m = 1$. This problem will be treated elsewhere[6].

In this paper we present a selection of tabulated values characterizing the function $\tau_a/\tau_b = f(m, \alpha)$, where τ is the turbidity determined at wavelengths λ_a and λ_b for the individual values of the relative refraction index $m = \mu_p/\mu_m$ and of the parameter $\alpha = \pi L/\lambda$; μ_p and μ_m respectively are the refractive indices of particles and of their

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medium. At the same time, it is demonstrated how the applicability of the turbidity ratio method and of the Tables can be extended to include systems with a different refractive index μ_m and with other pairs of wavelengths. The extension of the method to cover the NIR region will be reported in a forthcoming paper[7].

THE PRINCIPLE OF THE METHOD

The turbidity ratio method (*cf.*[1-3] and references cited therein) is based on the Lorenz–Mie theory of light scattering, according to which the turbidity of the system with N particles in 1 cm^3 can be expressed in terms of the equation[4]

$$\tau(\lambda, \alpha, m) = N\,R(\lambda, \alpha, m) = (\lambda^2/2\pi)\,N\,S(\alpha, m)\,, \tag{1}$$

where $S(\alpha, m) = \sum_1^\infty (|a_n|^2 + |b_n|^2)/(2n + 1)$, R is the scattering cross-section, a_n and b_n are complex functions of α and m (*cf.*[4]), $n = 1, 2, \ldots, \infty$. The particle size may be determined by the method of specific turbidity[4]:

$$(\tau/c)_0 = 0{\cdot}01(\varrho_s/\varrho_p)\,(3\pi/\lambda\alpha^3)\,S = 0{\cdot}01(\varrho_s/\varrho_p)\,(3\lambda^2/\pi^2L^3)\,S\,, \tag{2}$$

where c is concentration (g/100 g), ϱ_p and ϱ_s respectively are the densities of particles and of the system. The particle size may also be determined by using the method of the so-called wavelength exponent[8] based on the validity of the scattering law $\tau \sim \lambda^{-n}$ (its limiting case for $n = 4$ is the Rayleigh law). The turbidity ratio method stands close to the two techniques just described, as is shown below.

If we create the ratio of $(\tau/c)_0$ values determined for the pair of wavelengths λ_a and λ_b, we obtain

$$[(\tau/c)_0]_a/[(\tau/c)_0]_b = (\lambda_b/\lambda_a)\,(\alpha_b/\alpha_a)^3\,S_a/S_b = (\lambda_a/\lambda_b)^2\,S_a/S_b = (\tau_a/\tau_b)_0\,. \tag{3}$$

While the turbidity τ is a function of λ, α a m, the sum S depends only on α and m, which is very advantageous. The turbidity and sum values have been newly tabulated for the purposes of this work: though the procedure employed by Heller and co-workers has been observed in principle[9], a double-precision iteration has been used[10]. Assuming that within the range given by the individual pairs of wavelengths one need not bear in mind the dispersion of the relative refractive index m, the ratio of turbidities was calculated using the expression

$$\frac{\tau_a}{\tau_b} = \frac{\tau(\lambda, \alpha, m)}{\tau(\lambda\varkappa, \alpha/\varkappa, m)} = \frac{1}{\varkappa^2}\,\frac{S(\alpha, m)}{S(\alpha/\varkappa, m)} \tag{4}$$

or more generally (for the sequence of ratios τ_a/τ_b),

$$\frac{\tau_a}{\tau_b} = \frac{\tau_a(\lambda_0\varkappa^a, \alpha/\varkappa^a, m)}{\tau_b(\lambda_0\varkappa^b, \alpha/\varkappa^b, m)} = \frac{1}{\varkappa^{2(b-a)}}\,\frac{S_a(\alpha/\varkappa^a, m)}{S_b(\alpha/\varkappa^b, m)} = T_{ab}\,. \tag{5}$$

Here, the pair of the respective wavelengths is bound by the sequence condition $\lambda_a = \lambda_0\varkappa^a$ and $\lambda_b = \lambda_0\varkappa^b$, where $a = -2, -1, 0, 1, \ldots$, $b = -1, 0, 1, 2, \ldots$, $b - a = \delta = 1,2,3$; further, $\lambda_0 = \lambda_{a=0} = 326{\cdot}7$ nm $(= \Lambda_1/\varkappa\mu_w = 546{\cdot}1/\varkappa\mu_w = 409{\cdot}357/\varkappa = \Lambda_0/\mu_w = 435{\cdot}8/\mu_w)$; $\varkappa = \Lambda_1/\Lambda_0 = 546{\cdot}1/435{\cdot}8 = 1{\cdot}2531$ is the quotient of the given sequence introduced as yet another parameter for the tabulation of the turbidity ratios. Λ_1 and Λ_0 are wavelengths *in vacuo* corresponding to λ_1 and λ_0; $\mu_w = 1{\cdot}33398$ is the refractive index of water for the given conditions.

THE SCATTERING FUNCTION T_{ab} AND ITS UTILIZATION

The function $T_{ab} = \tau_a/\tau_b$ defined by (*4*) has been tabulated in the range of the following parameter values:

$\lambda_1 = 409{\cdot}357$ nm
$\alpha^* = 0{\cdot}05\ (0{\cdot}05)\ 6{\cdot}00\ (0{\cdot}10)\ 15{\cdot}00\ (0{\cdot}50)\ 70{\cdot}00\ (1{\cdot}00)\ 100\ (= \alpha/\varkappa^a)$
$m = 1{\cdot}001,\ 1{\cdot}005\ (0{\cdot}005)\ 1{\cdot}315$
$\varkappa^* = 1{\cdot}2531\ (= \varkappa),\ 1{\cdot}5703\ (= \varkappa^2),\ 1{\cdot}9677\ (= \varkappa^3)$

The full text can be found in volume D-2 of the series MACRO[11] published at the Institute of Macromolecular Chemistry of the Czechoslovak Academy of Sciences (where it is available to those interested). Table I contains a selection from these data in the following range:

$\lambda_1 = 409{\cdot}357$ nm
$\alpha^* = 0{\cdot}20\ (0{\cdot}20)\ 10{\cdot}00\ (1{\cdot}00)\ 20{\cdot}00\ (2{\cdot}00)\ 100\ (= \alpha/\varkappa^a)$
$m = 1{\cdot}001,\ 1{\cdot}050\ (0{\cdot}050)\ 1{\cdot}315$
$\varkappa^* = 1{\cdot}2531\ (= \varkappa),\ 1{\cdot}5703\ (= \varkappa^2),\ 1{\cdot}9677\ (= \varkappa^3)\,.$

PARTICLE SIZE DETERMINATION

The following procedure is employed in the particle size determination using Table I: Measurements at the chosen pairs of wave lengths $\Lambda_a = \lambda_a\mu_m$, $\Lambda_b = \lambda_b\mu_m$ (where μ_m is the refractive index of the medium) give us the respective turbidities, and thus also their ratios $T_{ab} = \tau_a/\tau_b$. Then we find out to which part of the tables the given ratio belongs according to $\varkappa^* = \varkappa^{b-a}$, *i.e.* $1{\cdot}2531\ (= \varkappa)$, $1{\cdot}5703\ (= \varkappa^2)$, $1{\cdot}9677\ (= \varkappa^3)$. In the column for the respective relative refractive index in Table I we find out into which range of T_{ab} values the measured τ_a/τ_b belongs and subsequently

TABLE I

Tabulation of the Scattering Function $T_{ab} = \tau_a/\tau_b$ Calculated Using Relation (*4*)

Parameter $\varkappa^* = 1{\cdot}2531(\varkappa)$, $1{\cdot}5703(\varkappa^2)$, $1{\cdot}9677(\varkappa^3)$; $\alpha^* = \alpha/\varkappa^a$; $\alpha = \pi L/\lambda_1$, where L is the diameter of the spherical particle, $\lambda_1 = 409{\cdot}357$ nm. The procedure for the particle size determination has been described in the text (*cf.* also Table III).

$\alpha^*\backslash m$	1·001	1·050	1·100	1·150	1·200	1·250	1·300
			$\varkappa^* = 1{\cdot}2531, \varkappa$				
0·20	2·451	2·453	2·455	2·457	2·458	2·460	2·462
0·40	2·410	2·416	2·423	2·429	2·436	2·442	2·448
0·60	2·343	2·356	2·368	2·381	2·393	2·406	2·418
0·80	2·257	2·274	2·291	2·308	2·326	2·343	2·359
1·00	2·156	2·175	2·193	2·210	2·228	2·244	2·260
1·20	2·050	2·065	2·078	2·091	2·102	2·112	2·120
1·40	1·946	1·954	1·959	1·962	1·964	1·963	1·962
1·60	1·853	1·853	1·851	1·847	1·843	1·839	1·839
1·80	1·779	1·775	1·772	1·770	1·772	1·782	1·803
2·00	1·729	1·729	1·734	1·746	1·769	1·805	1·853
2·20	1·704	1·715	1·735	1·767	1·809	1·855	1·893
2·40	1·700	1·725	1·760	1·800	1·835	1·850	1·829
2·60	1·708	1·743	1·780	1·805	1·805	1·767	1·695
2·80	1·720	1·754	1·776	1·770	1·732	1·667	1·596
3·00	1·727	1·749	1·747	1·715	1·663	1·610	1·575
3·20	1·723	1·729	1·707	1·668	1·627	1·601	1·588
3·40	1·710	1·702	1·674	1·643	1·621	1·605	1·576
3·60	1·692	1·677	1·654	1·636	1·620	1·590	1·534
3·80	1·673	1·660	1·647	1·633	1·608	1·559	1·491
4·00	1·658	1·652	1·644	1·627	1·588	1·529	1·463
4·20	1·649	1·649	1·641	1·614	1·566	1·505	1·438
4·40	1·645	1·648	1·635	1·599	1·546	1·483	1·413
4·60	1·644	1·646	1·625	1·584	1·528	1·464	1·395
4·80	1·643	1·641	1·614	1·570	1·514	1·450	1·377
5·00	1·642	1·634	1·604	1·559	1·503	1·436	1·346
5·20	1·638	1·627	1·595	1·550	1·492	1·412	1·306
5·40	1·633	1·620	1·589	1·543	1·476	1·384	1·282
5·60	1·628	1·614	1·584	1·534	1·458	1·365	1·275
5·80	1·623	1·611	1·579	1·522	1·443	1·355	1·257
6·00	1·619	1·608	1·573	1·512	1·433	1·341	1·212
6·20	1·617	1·606	1·567	1·503	1·422	1·311	1·169
6·40	1·615	1·603	1·561	1·496	1·404	1·280	1·154
6·60	1·614	1·600	1·556	1·486	1·384	1·263	1·154
6·80	1·612	1·597	1·551	1·474	1·368	1·257	1·127
7·00	1·611	1·594	1·545	1·463	1·359	1·241	1·075

TABLE I
(*Continued*)

$\alpha^*\backslash m$	1·001	1·050	1·100	1·150	1·200	1·250	1·300
			$\varkappa^* = 1{\cdot}2531$, $\varkappa$				
7·20	1·609	1·591	1·539	1·454	1·350	1·209	1·037
7·40	1·607	1·588	1·533	1·447	1·333	1·177	1·028
7·60	1·605	1·585	1·528	1·439	1·312	1·160	1·017
7·80	1·603	1·583	1·523	1·428	1·294	1·149	0·983
8·00	1·602	1·580	1·519	1·416	1·281	1·129	0·943
8·20	1·601	1·578	1·514	1·405	1·269	1·102	0·920
8·40	1·599	1·576	1·508	1·396	1·253	1·078	0·902
8·60	1·598	1·575	1·502	1·387	1·236	1·058	0·870
8·80	1·597	1·573	1·496	1·377	1·219	1·036	0·841
9·00	1·597	1·570	1·491	1·367	1·203	1·011	0·828
9·20	1·596	1·568	1·485	1·357	1·185	0·991	0·814
9·40	1·595	1·565	1·480	1·346	1·168	0·977	0·773
9·60	1·594	1·563	1·475	1·334	1·153	0·955	0·734
9·80	1·593	1·561	1·470	1·323	1·139	0·922	0·726
10·00	1·592	1·559	1·464	1·313	1·121	0·897	0·739
11·0	1·589	1·549	1·435	1·257	1·035	0·794	0·675
12·0	1·586	1·540	1·405	1·198	0·949	0·713	0·649
13·0	1·585	1·530	1·374	1·138	0·864	0·661	0·703
14·0	1·583	1·520	1·341	1·076	0·785	0·645	0·883
15·0	1·582	1·509	1·306	1·012	0·718	0·675	1·194
16·0	1·580	1·499	1·270	0·949	0·669	0·776	1·436
17·0	1·579	1·488	1·233	0·887	0·645	0·962	1·472
18·0	1·578	1·476	1·195	0·827	0·653	1·214	1·332
19·0	1·578	1·464	1·155	0·771	0·700	1·451	1·126
20·0	1·577	1·452	1·115	0·721	0·792	1·546	0·973
22·0	1·576	1·425	1·032	0·655	1·104	1·312	0·736
24·0	1·575	1·397	0·949	0·656	1·419	0·960	0·700
26·0	1·575	1·367	0·868	0·749	1·467	0·760	0·978
28·0	1·574	1·335	0·792	0·939	1·256	0·701	1·320
30·0	1·574	1·302	0·727	1·195	0·997	0·825	1·311
32·0	1·573	1·267	0·678	1·409	0·813	1·097	1·026
34·0	1·573	1·230	0·653	1·467	0·722	1·283	0·804
36·0	1·573	1·192	0·660	1·380	0·718	1·222	0·801
38·0	1·572	1·153	0·709	1·215	0·834	1·026	0·961
40·0	1·572	1·113	0·804	1·014	1·118	0·820	1·117
42·0	1·572	1·073	0·944	0·836	1·315	0·823	1·108
44·0	1·572	1·032	1·114	0·732	1·288	0·892	1·024

TABLE I

(Continued)

$\alpha^*\backslash m$	1·001	1·050	1·100	1·150	1·200	1·250	1·300
$\varkappa^* = 1{\cdot}2531,\ \varkappa$							
46·0	1·572	0·990	1·280	0·717	1·130	1·015	0·930
48·0	1·571	0·949	1·404	0·755	0·967	1·148	0·961
50·0	1·571	0·908	1·464	0·828	0·885	1·121	0·978
52·0	1·571	0·869	1·448	0·950	0·813	0·989	0·962
54·0	1·571	0·830	1·370	1·130	0·858	0·964	0·973
56·0	1·571	0·794	1·261	1·286	0·916	0·931	1·011
58·0	1·571	0·760	1·136	1·306	1·048	0·933	1·022
60·0	1·571	0·730	1·012	1·232	1·098	0·977	1·023
62·0	1·571	0·703	0·904	1·125	1·125	1·009	0·980
64·0	1·571	0·682	0·816	0·990	1·072	0·991	0·912
66·0	1·571	0·666	0·750	0·887	0·988	1·038	0·959
68·0	1·570	0·658	0·715	0·818	0·937	1·038	1·052
70·0	1·570	0·657	0·714	0·819	0·947	1·021	1·106
72·0	1·570	0·666	0·740	0·851	0·960	1·008	1·041
74·0	1·570	0·685	0·798	0·905	0·985	0·959	0·903
76·0	1·570	0·715	0·886	0·988	0·975	0·912	0·883
78·0	1·570	0·757	0·984	1·089	0·971	0·939	0·970
80·0	1·570	0·810	1·081	1·122	1·010	0·987	1·079
82·0	1·570	0·875	1·180	1·109	1·022	1·057	1·114
84·0	1·570	0·949	1·251	1·097	0·995	1·111	1·023
86·0	1·570	1·030	1·278	1·042	1·026	1·036	0·911
88·0	1·570	1·114	1·284	1·006	1·044	0·933	0·916
90·0	1·570	1·198	1·254	0·951	1·036	0·902	0·958
92·0	1·570	1·277	1·174	0·925	0·975	0·889	1·020
94·0	1·570	1·346	1·084	0·923	0·921	0·986	1·050
96·0	1·569	1·401	0·999	0·979	0·897	1·089	1·010
98·0	1·569	1·439	0·911	0·979	0·924	1·070	0·980
100·0	1·569	1·458	0·846	0·962	0·996	1·044	0·969
$x^* = 1{\cdot}5703,\ x^2$							
0·20	6·022	6·030	6·037	6·044	6·051	6·058	6·064
0·40	5·855	5·882	5·908	5·935	5·961	5·986	6·011
0·60	5·592	5·643	5·694	5·745	5·796	5·897	5·897
0·80	5·254	5·325	5·397	5·469	5·541	5·613	5·684
1·00	4·869	4·947	5·026	5·105	5·183	5·260	5·335
1·20	4·466	4·536	4·605	4·670	4·732	4·790	4·845
1·40	4·075	4·124	4·168	4·205	4·236	4·261	4·283

TABLE I
(*Continued*)

$\varkappa^* = 1{\cdot}5703, \varkappa^2$

$\alpha^*\backslash m$	1·001	1·050	1·100	1·150	1·200	1·250	1·300
1·60	3·723	3·746	3·762	3·771	3·776	3·780	3·787
1·80	3·429	3·434	3·434	3·433	3·437	3·454	3·490
2·00	3·206	3·207	3·212	3·228	3·263	3·323	3·411
2·20	3·055	3·070	3·100	3·150	3·224	3·316	3·412
2·40	2·969	3·009	3·070	3·149	3·235	3·307	3·339
2·60	2·933	2·999	3·080	3·159	3·215	3·225	3·180
2·80	2·928	3·010	3·088	3·139	3·145	3·100	3·018
3·00	2·935	3·016	3·073	3·086	3·051	2·980	2·886
3·20	2·940	3·007	3·034	3·015	3·957	2·868	2·741
3·40	2·934	2·981	2·982	2·941	3·862	2·741	2·569
3·60	2·917	2·943	2·924	2·865	2·759	2·607	2·425
3·80	2·890	2·900	2·867	2·787	2·660	2·504	2·351
4·00	2·859	2·857	2·810	2·715	2·585	2·448	2·322
4·20	2·827	2·817	2·759	2·658	2·538	2·415	2·277
4·40	2·797	2·781	2·716	2·619	2·506	2·373	2·198
4·60	2·770	2·749	2·683	2·589	2·471	2·314	2·117
4·80	2·746	2·722	2·658	2·563	2·431	2·254	2·047
5·00	2·724	2·700	2·637	2·536	2·388	2·197	1·970
5·20	2·704	2·683	2·620	2·508	2·344	2·133	1·887
5·40	2·689	2·670	2·603	2·478	2·295	2·067	1·826
5·60	2·676	2·660	2·585	2·444	2·245	2·015	1·793
5·80	2·667	2·650	2·564	2·408	2·202	1·982	1·751
6·00	2·661	2·640	2·541	2·374	2·170	1·946	1·672
6·20	2·655	2·627	2·518	2·347	2·141	1·888	1·584
6·40	2·649	2·613	2·500	2·324	2·103	1·823	1·528
6·60	2·641	2·600	2·479	2·300	2·058	1·772	1·495
6·80	2·632	2·585	2·463	2·273	2·016	1·736	1·442
7·00	2·622	2·574	2·448	2·244	1·983	1·695	1·371
7·20	2·613	2·564	2·432	2·218	1·952	1·641	1·311
7·40	2·604	2·556	2·416	2·195	1·915	1·589	1·271
7·60	2·597	2·548	2·401	2·171	1·875	1·546	1·217
7·80	2·592	2·541	2·387	2·146	1·837	1·502	1·145
8·00	2·587	2·534	2·372	2·119	1·801	1·447	1·089
8·20	2·583	2·527	2·358	2·093	1·762	1·395	1·063
8·40	2·580	2·520	2·342	2·066	1·722	1·358	1·032
8·60	2·576	2·513	2·326	2·039	1·686	1·327	0·967
8·80	2·573	2·506	2·310	2·012	1·656	1·283	0·899
9·00	2·569	2·500	2·294	1·988	1·625	1·225	0·860

TABLE I

(*Continued*)

$\alpha^*\backslash m$	1·001	1·050	1·100	1·150	1·200	1·250	1·300
$\varkappa^* = 1\cdot5703, \varkappa^2$							
9·20	2·566	2·491	2·279	1·966	1·586	1·175	0·838
9·40	2·562	2·484	2·265	1·942	1·544	1·140	0·793
9·60	2·559	2·476	2·252	1·916	1·506	1·104	0·740
9·80	2·555	2·470	2·239	1·888	1·472	1·058	0·711
10·00	2·551	2·464	2·225	1·861	1·438	1·015	0·700
11·0	2·539	2·437	2·148	1·732	1·264	0·825	0·569
12·0	2·530	2·407	2·074	1·601	1·096	0·683	0·478
13·0	2·521	2·381	1·996	1·471	0·937	0·581	0·469
14·0	2·515	2·352	1·918	1·342	0·798	0·511	0·568
15·0	5·509	2·325	1·837	1·215	0·684	0·482	0·778
16·0	2·505	2·297	1·755	1·093	0·593	0·514	1·000
17·0	2·501	2·268	1·672	0·978	0·530	0·606	1·232
18·0	2·500	2·238	1·587	0·870	0·500	0·751	1·471
19·0	2·495	2·208	1·503	0·772	0·501	0·960	1·489
20·0	2·492	2·177	1·418	0·687	0·529	1·203	1·381
22·0	2·489	2·112	1·251	0·559	0·701	1·428	1·148
24·0	2·485	2·043	1·090	0·501	0·990	1·434	0·796
26·0	2·483	1·972	0·940	0·519	1·345	1·111	0·854
28·0	2·481	1·897	0·806	0·613	1·481	0·919	0·878
30·0	2·479	1·820	0·691	0·781	1·391	0·794	0·934
32·0	2·477	1·741	0·600	0·998	1·198	0·881	0·882
34·0	2·476	1·660	0·538	1·238	0·977	0·881	0·945
36·0	2·475	1·578	0·506	1·436	0·873	0·916	1·047
38·0	2·474	1·495	0·508	1·493	0·806	0·854	1·189
40·0	2·473	1·412	0·546	1·423	0·898	0·898	1·098
42·0	2·473	1·330	0·620	1·234	0·912	1·022	0·901
44·0	2·472	1·248	0·726	1·056	0·908	1·164	0·805
46·0	2·471	1·167	0·861	0·950	0·894	1·157	0·826
48·0	2·471	1·089	1·013	0·871	0·874	1·123	0·973
50·0	2·470	1·013	1·168	0·851	0·960	0·922	1·119
52·0	2·470	0·941	1·311	0·851	0·954	0·801	1·090
54·0	2·470	0·872	1·419	0·874	1·145	0·782	1·037
56·0	2·469	0·808	1·476	0·927	1·164	0·871	0·961
58·0	2·469	0·748	1·477	0·924	1·213	0·941	0·975
60·0	2·468	0·694	1·416	0·920	1·101	1·113	0·957
62·0	2·468	0·646	1·315	0·894	0·958	1·059	0·951
64·0	2·468	0·605	1·195	0·874	0·879	1·052	0·887

TABLE I
(*Continued*)

$\alpha^*\backslash m$	1·001	1·050	1·100	1·150	1·200	1·250	1·300
				$\varkappa^* = 1{\cdot}5703,\ \varkappa^2$			
66·0	2·468	0·570	1·073	0·932	0·794	0·988	0·949
68·0	2·467	0·543	0·974	0·946	0·819	0·998	1·043
70·0	2·467	0·524	0·906	1·020	0·871	0·955	1·123
72·0	2·467	0·512	0·866	1·103	0·987	0·989	1·088
74·0	2·467	0·510	0·854	1·154	1·069	0·919	0·935
76·0	2·466	0·515	0·861	1·233	1·110	0·896	0·874
78·0	2·466	0·530	0·873	1·185	1·061	0·905	0·895
80·0	2·466	0·554	0·889	1·107	1·062	0·971	0·966
82·0	2·466	0·586	0·903	1·019	1·025	1·058	0·998
84·0	2·466	0·627	0·910	0·919	0·947	1·148	1·030
86·0	2·465	0·676	0·919	0·862	0·983	1·089	1·017
88·0	2·465	0·733	0·922	0·801	0·973	0·990	1·018
90·0	2·465	0·797	0·921	0·802	0·977	0·925	1·029
92·0	2·465	0·867	0·921	0·839	0·942	0·865	0·976
94·0	2·465	0·941	0·911	0·886	0·903	0·909	0·915
96·0	2·464	1·018	0·899	1·002	0·907	0·966	0·894
98·0	2·464	1·096	0·902	1·028	0·912	1·013	0·954
100·0	2·464	1·173	0·911	1·090	0·981	1·013	1·028
				$x^* = 1{\cdot}9677,\ x^3$			
0·20	14·814	14·837	14·859	14·881	14·903	14·924	14·944
0·40	14·303	14·384	14·467	14·548	14·628	14·707	14·784
0·60	13·503	13·659	13·818	13·977	14·135	14·293	14·448
0·80	12·485	12·705	12·931	13·157	13·384	13·611	13·838
1·00	11·339	11·587	11·842	12·097	12·352	12·605	12·856
1·20	10·153	10·387	10·622	10·851	11·073	11·287	11·495
1·40	9·014	9·198	9·372	9·532	9·678	9·811	9·934
1·60	7·991	8·110	8·212	8·297	8·371	8·441	8·517
1·80	7·128	7·194	7·247	7·294	7·349	7·427	7·544
2·00	6·446	6·490	5·535	6·598	6·694	6·836	7·032
2·20	5·943	5·996	6·071	6·181	6·329	6·509	6·693
2·40	5·594	5·679	5·794	5·937	6·087	6·210	6·262
2·60	5·367	5·485	5·622	5·752	5·841	5·855	5·784
2·80	5·223	5·360	5·487	5·571	5·585	5·531	5·440
3·00	5·130	5·266	5·363	5·399	5·378	5·325	5·265
3·20	5·063	5·184	5·253	5·268	5·248	5·205	5·113
3·40	5·007	5·113	5·167	5·180	5·155	5·065	4·862

TABLE I
(*Continued*)

$\alpha^*\backslash m$	1·001	1·050	1·100	1·150	1·200	1·250	1·300
			$\varkappa^* = 1{\cdot}9677,\ \varkappa^3$				
3·60	4·957	5·054	5·105	5·109	5·040	4·857	4·553
3·80	4·913	5·008	5·053	5·023	4·882	4·619	4·272
4·00	4·876	4·969	4·992	4·909	4·701	4·392	4·020
4·20	4·843	4·928	4·915	4·776	4·519	4·174	3·756
4·40	4·813	4·879	4·823	4·635	4·338	3·953	3·508
4·60	4·781	4·818	4·720	4·493	4·162	3·757	3·334
4·80	4·742	4·748	4·614	4·356	4·008	3·615	3·231
5·00	4·696	4·673	4·511	4·237	3·890	3·517	3·127
5·20	4·644	4·598	4·420	4·141	3·802	3·422	2·994
5·40	4·590	4·530	4·344	4·066	3·721	3·314	2·866
5·60	4·537	4·471	4·282	4·000	3·637	3·212	2·765
5·80	4·489	4·421	4·230	3·937	3·557	3·124	2·648
6·00	4·448	4·379	4·184	3·878	3·486	3·027	2·487
6·20	4·412	4·344	4·141	3·823	3·412	2·901	2·328
6·40	4·381	4·313	4·103	3·771	3·321	2·769	2·222
6·60	4·355	4·285	4·068	3·712	3·222	2·666	2·150
6·80	4·332	4·261	4·033	3·646	3·131	2·588	2·050
7·00	4·312	4·240	3·995	3·578	3·056	2·504	1·929
7·20	4·295	4·221	3·954	3·515	2·986	2·406	1·831
7·40	4·281	4·201	3·912	3·458	2·911	2·313	1·762
7·60	4·268	4·179	3·871	3·403	2·835	2·238	1·670
7·80	4·256	4·155	3·832	3·348	2·765	2·160	1·549
8·00	4·244	4·131	3·795	3·295	2·699	2·064	1·445
8·20	4·231	4·108	3·759	3·243	2·627	1·965	1·384
8·40	4·217	4·086	3·725	3·192	2·550	1·888	1·327
8·60	4·202	4·065	3·692	3·139	2·478	1·825	1·236
8·80	4·187	4·046	3·659	3·086	2·414	1·750	1·146
9·00	4·173	4·028	3·627	3·035	2·352	1·664	1·090
9·20	4·160	4·011	3·596	2·987	2·284	1·589	1·045
9·40	4·149	3·995	3·565	2·938	2·213	1·530	0·965
9·60	4·139	3·980	3·536	2·887	2·149	1·464	0·877
9·80	4·129	3·965	3·506	2·836	2·089	1·380	0·827
10·00	4·121	3·952	3·476	2·785	2·024	1·303	0·808
11·0	4·090	3·885	3·318	2·533	1·718	1·023	0·611
12·0	4·060	3·815	3·167	2·300	1·433	0·791	0·484
13·0	4·034	3·756	3·018	2·061	1·184	0·635	0·429
14·0	4·017	3·696	2·863	1·840	0·965	0·522	0·473
15·0	4·001	3·635	2·713	1·624	0·791	0·463	0·576

TABLE I

(*Continued*)

$\alpha^*\backslash m$	1·001	1·050	1·100	1·150	1·200	1·250	1·300
$\varkappa^* = 1{\cdot}9677,\ \varkappa^3$							
16·0	3·987	3·578	2·560	1·424	0·653	0·455	0·716
17·0	3·976	3·518	2·407	1·238	0·558	0·490	0·823
18·0	3·967	3·458	2·257	1·070	0·497	0·580	0·878
19·0	3·958	3·398	2·107	0·921	0·470	0·684	0·951
20·0	3·951	3·336	1·960	0·792	0·471	0·796	0·961
22·0	3·939	3·209	1·677	0·601	0·550	0·919	1·021
24·0	3·930	3·078	1·414	0·497	0·704	0·955	1·091
26·0	3·922	2·943	1·176	0·474	0·869	0·999	1·274
28·0	3·916	2·805	0·968	0·513	0·958	1·049	1·233
30·0	3·911	2·663	0·796	0·598	0·971	1·183	1·049
32·0	3·907	2·519	0·660	0·707	0·988	1·306	0·848
34·0	3·903	2·375	0·563	0·824	1·034	1·225	0·733
36·0	3·900	2·229	0·503	0·926	1·098	1·041	0·756
38·0	3·897	2·085	0·478	0·988	1·198	0·848	0·887
40·0	3·895	1·942	0·485	1·004	1·335	0·731	0·952
42·0	3·893	1·802	0·520	0·993	1·298	0·771	0·988
44·0	3·891	1·666	0·575	0·994	1·133	0·809	1·064
46·0	3·889	1·534	0·644	1·039	0·955	0·879	1·026
48·0	3·888	1·407	0·720	1·112	0·828	0·977	1·181
50·0	3·887	1·287	0·796	1·189	0·767	1·000	1·062
52·0	3·885	1·173	0·866	1·255	0·729	1·011	0·950
54·0	3·884	1·067	0·925	1·295	0·794	1·085	0·822
56·0	3·883	0·969	0·970	1·297	0·855	1·084	0·818
58·0	3·882	0·879	1·000	1·212	0·924	1·135	0·848
60·0	3·881	0·798	1·016	1·077	0·965	1·090	0·968
62·0	3·880	0·727	1·020	0·934	0·983	0·998	1·037
64·0	3·880	0·664	1·017	0·828	1·015	0·857	1·026
66·0	3·879	0·611	1·016	0·777	1·054	0·798	1·054
68·0	3·878	0·568	1·025	0·743	1·078	0·841	1·069
70·0	3·878	0·534	1·051	0·744	1·107	0·868	1·073
72·0	3·877	0·509	1·097	0·779	1·130	0·999	1·016
74·0	3·876	0·493	1·159	0·843	1·119	1·016	0·903
76·0	3·876	0·485	1·224	0·910	1·042	0·962	0·844
78·0	3·875	0·485	1·276	0·967	0·955	1·028	0·860
80·0	3·875	0·493	1·304	0·976	0·890	1·012	0·937
82·0	3·874	0·507	1·304	0·990	0·829	1·096	1·015
84·0	3·874	0·527	1·276	1·015	0·794	1·110	1·036

TABLE I
(Continued)

α^*/m	1·001	1·050	1·100	1·150	1·200	1·250	1·300
			$x^* = 1{\cdot}9677,\ x^3$				
86·0	3·873	0·552	1·226	1·017	0·849	0·977	1·007
88·0	3·873	0·582	1·161	1·037	0·903	0·919	1·032
90·0	3·872	0·615	1·088	1·058	0·986	0·841	1·046
92·0	3·872	0·651	1·008	1·090	1·017	0·850	1·029
94·0	3·871	0·688	0·927	1·121	1·023	0·895	0·964
96·0	3·871	0·727	0·851	1·151	0·985	0·955	0·863
98·0	3·871	0·766	0·791	1·125	1·026	0·975	0·880
100·0	3·870	0·804	0·752	1·084	1·015	1·004	0·924

read off the corresponding α in the first column (after an interpolation, if needed). For the basic ratios (τ_0/τ_1, τ_0/τ_2, τ_0/τ_3) the value thus found is identical with the actual $\alpha = \pi L/\lambda$, where $\lambda = 409{\cdot}357$ nm. In all the other cases other scales α are valid, the general representation of which can be seen in Eq. (*5*) and in Tables II and III, which summarize conditions for all combinations that may be contemplated in practice. It can be seen that α^* read off from Table I gradually assumes the values $\alpha^* = \alpha/\varkappa^a$, so that the actual α is obtained by calculation from $\alpha = \alpha^*\varkappa^a$.

The Limiting Value T^0_{ab}

Table III also gives the limiting value T^0_{ab} for $\alpha \to 0$ calculated from the expression (expressed in the scale α^*)

$$T^0_{ab} = (\tau_a/\tau_b)_{\alpha\to 0} = \varkappa^{4b-3a}\,. \tag{6}$$

Equation (*6*) may be derived as follows: According to the Rayleigh theory, the expression

$$(\tau/\phi)_0 = (4\pi\alpha^3/\lambda)\,\xi^2\,, \tag{7}$$

is valid for small particles; here, ϕ is the volume fraction of particles ($c = 10^{-2}\phi\,.\,\cdot\,\varrho_s/\varrho_p'$), $\xi = (m^2 - 1)/(m^2 + 2)$. If, according to the Lorenz–Mie theory, we put $(\tau/\phi)_0 = (3\pi/\alpha^3)\,S$ and substitute into Eq. (*7*), we obtain

$$S = (4/3)\,\alpha^6\xi^2\,. \tag{8}$$

The creation of S_a/S_b gives ($\xi = \text{const}$)

$$S_a/S_b = (\alpha_a/\alpha_b)^6 = (\lambda_b/\lambda_a)^6 = \varkappa^{6(b-a)}\,; \tag{9}$$

and substitution for S_a/S_b into Eq. (*5*) gives

$$T^0_{ab} = (\tau_a/\tau_b)_{\alpha \to 0} = \varkappa^{4(b-a)}\,. \tag{10}$$

It should be borne in mind, of course, that T^0_{ab} is thus expressed in the scale $\alpha/\varkappa^a$ and not in the scale α (*cf.* Eq. (*5*) and Table III); for this reason, Eq. (*10*) must be multiplied by the factor $\varkappa^a$, which yields Eq. (*6*).

Particles in a Medium with the Refractive Index $\mu_m \neq \mu_w$

Data in Table I have been calculated for the basic wavelength $\lambda_1 = 409{\cdot}357$ nm ($= \Lambda_1/\mu_w$ where $\Lambda_1 = 546{\cdot}1$ nm), from which wavelengths λ_a and λ_b (and the respective Λ_a and Λ_b) have been derived, at which τ_a/τ_b values are determined (Table III). If the size of particles suspended in a medium having a refractive index different from μ_w is to be determined, two procedures may be employed:

a) Wavelengths Λ_a and Λ_b are adjusted so as to observe the respective wavelengths λ_a and λ_b for which the data have been tabulated (Tables II and III). If the refractive

TABLE II

The Sequence of Wavelengths $\lambda(\Lambda)$ and of the Parameters $\varkappa_k$ (Exponent k = 1 to 17) for the Determination of T_{ab} and T^0_{ab} According to Table III

Wavelengths, nm		Parameter $\varkappa^k$	
λ_{-2} = 208·0	Λ_{-2} = 277·5	$\varkappa^{-2}$ = 0·6368	$\varkappa^9$ = 7·6185
λ_{-1} = 260·7	Λ_{-1} = 347·8	$\varkappa^{-1}$ = 0·7980	$\varkappa^{10}$ = 9·5468
λ_0 = 326·7	Λ_0 = 435·8	$\varkappa^0$ = 1·0000	$\varkappa^{11}$ = 11·9631
λ_1 = 409·3	Λ_1 = 546·1	$\varkappa^1$ = 1·2531	$\varkappa^{12}$ = 14·9909
λ_2 = 513·0	Λ_2 = 684·3	$\varkappa^2$ = 1·5703	$\varkappa^{13}$ = 18·7851
λ_3 = 642·8	Λ_3 = 857·5	$\varkappa^3$ = 1·9677	$\varkappa^{14}$ = 23·5397
λ_4 = 805·5	Λ_4 = 1 074	$\varkappa^4$ = 2·4657	$\varkappa^{15}$ = 29·4975
λ_5 = 1 009	Λ_5 = 1 346	$\varkappa^5$ = 3·0898	$\varkappa^{16}$ = 36·9634
λ_6 = 1 265	Λ_6 = 1 687	$\varkappa^6$ = 3·8718	$\varkappa^{17}$ = 46·3188
λ_7 = 1 585	Λ_7 = 2 114	$\varkappa^7$ = 4·8518	
λ_8 = 1 986	Λ_8 = 2 649	$\varkappa^8$ = 6·0797	

index of the medium is μ_m, the required wavelengths Λ'_a and Λ'_b obey the relation

$$\Lambda' = \Lambda\mu_m/\mu_w\,. \qquad (11)$$

b) The tables are adjusted for new pairs of wavelengths (*cf.* the following paragraph).

TABLE III

Determination of Values $T_{ab} = \tau_a/\tau_b$ and $T^0_{ab} = \varkappa^{4b-3a} = [T_{ab}]_{\alpha \to 0}$
Numerical λ and $\varkappa$ values see Table II.

T_{ab}	$(\varkappa^*)^{-1}$	$\varkappa^*$	α^*	α	T^0_{ab}
τ_a/τ_b	λ_a/λ_b	$\varkappa^{b-a}$	$\alpha/\varkappa^a$	$\alpha^*\varkappa^a$	$\varkappa^{4b-3a}$
τ_{-2}/τ_{-1}	$\lambda_{-2}/\lambda_{-1}$	$\varkappa$	$\alpha\varkappa^2$	$\alpha^*/\varkappa^2$	$\varkappa^2$
τ_{-2}/τ_0	λ_{-2}/λ_0	$\varkappa^2$	$\alpha\varkappa^2$	$\alpha^*/\varkappa^2$	$\varkappa^6$
τ_{-2}/τ_1	λ_{-2}/λ_1	$\varkappa^3$	$\alpha\varkappa^2$	$\alpha^*/\varkappa^2$	$\varkappa^{10}$
τ_{-1}/τ_0	λ_{-1}/λ_0	$\varkappa$	$\alpha\varkappa$	$\alpha^*/\varkappa$	$\varkappa^3$
τ_{-1}/τ_1	λ_{-1}/λ_1	$\varkappa^2$	$\alpha\varkappa$	$\alpha^*/\varkappa$	$\varkappa^7$
τ_{-1}/τ_2	λ_{-1}/λ_2	$\varkappa^3$	$\alpha\varkappa$	$\alpha^*/\varkappa$	$\varkappa^{11}$
τ_0/τ_1	λ_0/λ_1	$\varkappa$	α	α^*	$\varkappa^4$
τ_0/τ_2	λ_0/λ_2	$\varkappa^2$	α	α^*	$\varkappa^8$
τ_0/τ_3	λ_0/λ_3	$\varkappa^3$	α	α^*	$\varkappa^{12}$
τ_1/τ_2	λ_1/λ_2	$\varkappa$	$\alpha/\varkappa$	$\alpha^*\varkappa$	$\varkappa^5$
τ_1/τ_3	λ_1/λ_3	$\varkappa^2$	$\alpha/\varkappa$	$\alpha^*\varkappa$	$\varkappa^9$
τ_1/τ_4	λ_1/λ_4	$\varkappa^3$	$\alpha/\varkappa$	$\alpha^*\varkappa$	$\varkappa^{13}$
τ_2/τ_3	λ_2/λ_3	$\varkappa$	$\alpha/\varkappa^2$	$\alpha^*\varkappa^2$	$\varkappa^6$
τ_2/τ_4	λ_2/λ_4	$\varkappa^2$	$\alpha/\varkappa^2$	$\alpha^*\varkappa^2$	$\varkappa^{10}$
τ_2/τ_5	λ_2/λ_5	$\varkappa^3$	$\alpha/\varkappa^2$	$\alpha^*\varkappa^2$	$\varkappa^{14}$
τ_3/τ_4	λ_3/λ_4	$\varkappa$	$\alpha/\varkappa^3$	$\alpha^*\varkappa^3$	$\varkappa^7$
τ_3/τ_5	λ_3/λ_5	$\varkappa^2$	$\alpha/\varkappa^3$	$\alpha^*\varkappa^3$	$\varkappa^{11}$
τ_3/τ_6	λ_3/λ_6	$\varkappa^3$	$\alpha/\varkappa^3$	$\alpha^*\varkappa^3$	$\varkappa^{15}$
τ_4/τ_5	λ_4/λ_5	$\varkappa$	$\alpha/\varkappa^4$	$\alpha^*\varkappa^4$	$\varkappa^8$
τ_4/τ_6	λ_4/λ_6	$\varkappa^2$	$\alpha/\varkappa^4$	$\alpha^*\varkappa^4$	$\varkappa^{12}$
τ_4/τ_7	λ_4/λ_7	$\varkappa^3$	$\alpha/\varkappa^4$	$\alpha^*\varkappa^4$	$\varkappa^{16}$
τ_5/τ_6	λ_5/λ_6	$\varkappa$	$\alpha/\varkappa^5$	$\alpha^*\varkappa^5$	$\varkappa^9$
τ_5/τ_7	λ_5/λ_7	$\varkappa^2$	$\alpha/\varkappa^5$	$\alpha^*\varkappa^5$	$\varkappa^{13}$
τ_5/τ_8	λ_5/λ_8	$\varkappa^3$	$\alpha/\varkappa^5$	$\alpha^*\varkappa^5$	$\varkappa^{17}$

Selection of Other Pairs of Wavelengths

The data summarized in Table I may also be used in those cases where it is necessary to change the pairs of wavelengths (other dispersing medium, absorption in the given range of the spectrum, *etc.*). The $(\tau/c)_0$ values determined for the basic wavelength λ may be recalculated[4] to another wavelength λ' by simple multiplication: $(\tau/c)'_0 = (\tau/c)_0 (\lambda/\lambda')$; $\alpha = \pi L/\lambda'$. Hence, for data of the new table the equation

$$T'_{ab} = \tau'_a/\tau'_b = \tau_a(\lambda/\lambda')/\tau_b(\lambda/\lambda') = T_{ab} \qquad (12)$$

is automatically valid. The numerical T'_{ab} values are identical with T_{ab} in the original table, but for α we have $\alpha = \pi L/\lambda'$ (data are related to a new base λ'). If the quotient $\varkappa$ is retained for the new series of λ'_a, λ'_b values, Table I with the base λ' may be used in full extent. If it became necessary to respect the dispersion of the refractive index μ_m, procedure (*a*) from the preceding paragraph (Eq. (*11*)) should be used; differences between the values thus obtained are of course insignificant as a rule. Correction for the dispersion of the relative refractive index m would be much more important, but so far it has not been available in a suitable form. Since the effect of dispersion is the stronger the more remote are the wavelengths used, it is desirable, in the turbidity measurements, to restrict oneself in each case to four terms of the series at most ($\delta = 1, 2, 3$).

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REFERENCES

1. Sedláček B.: Rep. Inst. Macromol. Chem., Prague 1962. Preprints, IUPAC International Symposium on Macromolecules, Prague 1965.
2. Sedláček B.: This Journal *32*, 1374 (1967).
3. Sedláček B.: This Journal *32*, 1398 (1967).
4. Heller W., Pangonis W. J.: J. Chem. Phys. *26*, 498 (1957).
5. Heller W.: J. Chem. Phys. *26*, 920 (1957).
6. Sedláček B.: Unpublished results.
7. Sedláček B., Jokl J.: Unpublished results.
8. Heller W., Bhatnagar, Nakagaki M.: J. Chem. Phys. *62*, 1163 (1962).
9. Pangonis W. J., Heller W.: *Angular Scattering Functions for Spherical Particles.* Wayne State University Press, Detroit 1960.
10. Verner B., Bárta M., Sedláček B.: $(\tau/c)_0$, *K, R, S Functions.* Series MACRO, Vol. D-1, Inst. Macromol. Chem., Prague 1976.
11. Sedláček B., Verner B., Bárta M., Zimmermann K.: *Turbidity Ratios.* Series MACRO, Vol. D-2. Inst. Macromol. Chem., Prague 1978.

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